International Conference on Multivariate Approximation

Arcadeon Hagen
September 24–27, 2011

Martin D. Buhmann, Gießen
Kurt Jetter, Stuttgart–Hohenheim
Joachim Stöckler, Dortmund

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Technische Universität Dortmund

Preface

This International Conference on Multivariate Approximation continues our series of conferences that began in 1994. All earlier meetings were held in the conference center "Haus Bommerholz" of the Technische Universität Dortmund. This year the new venue was chosen to be "Arcadeon" in Hagen - a well-known conference center in the Westphalia region.

Particular key topics of the conference are

- Kernel based methods and radial basis functions
- Multivariate polynomial and spline approximation
- Multivariate subdivision and wavelets
- Nonlinear approximation and compressed sensing

These and further topics are presented in 9 invited talks and 25 contributed talks. As most participants stay in the conference hotel and the Landhotel nearby, there will be ample time for discussions during the conference days.

We gratefully acknowledge the financial support of this conference by the Deutsche Forschungsgemeinschaft and by the Technische Universität Dortmund.

We thank all participants for their contributions and hope, especially, that fruitful and productive discussions will make this conference a success.

We started organizing this conference together with our colleague and friend Werner Haussmann who died unexpectedly last year. He also was co-organizer of all previous conferences in this series. We revere him for his great enthusiasm in supporting the international community of mathematicians, especially in areas of Approximation Theory.

Dortmund, September 2011

The Organizers:

Martin Buhmann
Kurt Jetter
Joachim Stöckler

Participants

Severin Bannert	University of Vienna
Guenter Baszenski	FH Dortmund
Peter Binev	University of South Carolina
Dietrich Braess	Ruhr-Universität Bochum
Johann Brauchart	University of New South Wales
Martin Buhmann	Justus-Liebig-Universität Giessen
Wolfgang zu Castell	Helmholtz Zentrum München
Maria Charina-Kehrein	TU Dortmund
Oleg Davydov	University of Strathclyde Glasgow
Stefano De Marchi	University of Padova
Franz-Juergen Delvos	University of Siegen
Martin Ehler	Helmholtz Zentrum München
Frank Filbir	Helmholtz-Zentrum München
Heiner Gonska	Universität Duisburg-Essen
Karlheinz Groechenig	University of Vienna
Michael Hagen	Helmholtz-Zentrum München
Kurt Jetter	Universität Hohenheim
Emily King	Universität Osnabrück
Stefan Kunis	Universität Osnabrück
Gitta Kutyniok	Technische Universität Berlin
Kyoung-Yong Lee	ISD Dortmund
Tom Lyche	University of Oslo
Carla Manni	Universitá di Roma "Tor Vergata"
Marie-Laurence Mazure	University Joseph Fourier
H. Michael Möller	TU Dortmund
Bernd Mulansky	TU Clausthal
Georg Muntingh	Centre of Mathematics for Applications Oslo
Mikhail Noskov	Siberian Federal University
Peter Oswald	Jacobs University Bremen
Maryam Pazouki	Universität Göttingen
Francesca Pelosi	Universitá di Roma "Tor Vergata"
Emilio Porcu	University of Castilla - La Mancha

Vladimir PROTASOV	Moscow State University
Christophe RABUT	INSA
Holger RAUHUT	University of Bonn
Manfred REIMER	TU Dortmund
Hermann RENDER	University College Dublin
Maria Daniela RUSU	Universität Duisburg-Essen
Tomas SAUER	JLU Gießen
Larry SCHUMAKER	Vanderbilt University
Alexei SHADRIN	University of Cambridge
Katrin SIEMKO	TU Dortmund
Michael SKRZIPEK	Fernuniversität Hagen
Hendrik SPELEERS	Katholieke Universiteit Leuven
Tobias SPRINGER	TU Dortmund
Elena Dorina STANILA	Universität Duisburg-Essen
Gabriele STEIDL	University of Kaiserslautern
Joachim STÖCKLER	Technische Universität Dortmund
Gancho TACHEV	University of Architecture Civil Engineering and Geodesy, Sofia
Nelly VILLAMIZAR	University of Oslo
Michael WOZNICZKA	Dassault Systèmes Hannover
Georg ZIMMERMANN	Universität Hohenheim

Conference Program

Saturday, September 24

08:45–09:00	*Welcome and Opening Remarks*

1st Morning Session:

09:00–09:25	Larry L. Schumaker, Vanderbilt University, USA *Splines on triangulations with hanging vertices*
09:25–09:50	Tom Lyche, University of Oslo, Norway *Local refinable multivariate splines*
09:50–10:15	Oleg Davydov, University of Strathclyde, UK *Hierarchical Riesz bases for Sobolev spaces on polygonal domains*
10:15–10:40	Nelly Villamizar, University of Oslo, Norway *Multivariate polynomial and spline approximation:* *Dimension of triangular spline spaces*
10:40–11:10	**Coffee break**

2nd Morning Session:

11:10–12:00[1]	Gitta Kutyniok, Technische Universität Berlin, Germany *Shearlets and sparse approximation*
12:00–12:25	Martin Ehler, Helmholtz Zentrum München, Germany *Fusion frames*
12:25–12:50	Tobias Springer, Technische Universität Dortmund, Germany *Cluster analysis and frame potential*
12:50	**Lunch**

Afternoon Session:

15:30–16:20	Emilio Porcu, Universidad de Castilla - La Mancha, Spain *Compactly supported correlations: a statistical view*
16:20–16:45	Peter Oswald, Jacobs University Bremen, Germany *Ortho-projectors onto linear spline spaces over* *arbitrary triangulations: L_∞-bounds*
16:45–17:10	Mikhail Noskov, Siberian Federal University, Russia *On error estimates of cubature formulas exact for Haar polynomials*

[1] Times in bold-face represent an invited keynote presentation

Sunday, September 25

1st Morning Session:

08:30–09:20	Holger Rauhut, Universität Bonn, Germany *Compressive sensing, structured random matrices and recovery of functions in high dimensions*
09:20–09:45	Peter Binev, University of South Carolina, USA *Approximation in high dimensions*
09:45–10:15	Stefan Kunis, Universität Osnabrück, Germany *Sparse FFTs and photoacoustic tomography*
10:15–10:40	**Coffee break**

2nd Morning Session:

10:40–11:30	Wolfgang zu Castell, Helmholtz Zentrum München, Germany *Kernel based approximation: from data to special functions*
11:30–11:55	Frank Filbir, Helmholtz Zentrum München, Germany *Localized kernels and detection of singularities*
11:55–12:20	Emily King, Universität Osnabrück *Image inpainting via ℓ^1-minimization and thresholding*
12:30	**Lunch**

Afternoon Excursion:

By Wuppertal's "Schwebebahn" to the von der Heidt museum

Evening:

Barbecue Dinner at the Arcadeon

Monday, September 26

1st Morning Session:

08:30–09:20	Carla Manni, Università di Roma “Tor Vergata”, Italy *Non standard discretization techniques* *in isogeometric analysis*
09:20–09:45	Hendrik Speleers, Katholieke Universiteit Leuven, Belgium *Normalized hierarchical B-splines and isogeometric analysis*
09:45–10:10	Francesca Pelosi, Università di Roma “Tor Vergata”, Italy *Local refinements in isogeometric analysis* *with generalized B-splines*
10:10–10:40	**Coffee break**

2nd Morning Session:

10:40–11:30	Alexei Shadrin, Cambridge University, UK *Orthogonal spline-projector and related questions*
11:30–11:55	Marie-Laurence Mazure, Université Joseph Fourier, France *Chebyshevian splines and Chebyshevian subdivision schemes*
11:55–12:40	Michael Möller, Technische Universität Dortmund, Germany *The multivariate moment problem*
12:30	**Lunch**

Afternoon Session:

15:30–16:20	Tomas Sauer, Justus Liebig Universität Gießen, Germany *Divided differences by interpolation*
16:20–16:45	Georg Muntingh, University of Oslo, Norway *Divided differences of multivariate implicit functions*
16:45–17:10	**Coffee break**
17:10–17:35	Gabriele Steidl, Universität Kaiserslautern, Germany *Quadrature rules, discrepancies and their relations* *to halftoning on the torus and the sphere*
17:35–18:00	Johann Brauchart, University of New South Wales, Australia *Construction of point configuration on hyper spheres*

Tuesday, September 27

1st Morning Session:

08:30–09:20	Vladimir Protasov, Moscow State University, Russia *Methods of the exact joint spectral radius computation*
09:20–09:45	Georg Zimmermann, Universität Hohenheim, Germany *Scalar Multivariate Subdivision Schemes with scaling factor* > 2 *and box splines*
09:45–10:10	Maria Charina, Technische Universität Dortmund, Germany *Vector subdivision methods*
10:10–10:40	**Coffee break**

2nd Morning Session:

10:40–11:30	Hermann Render, University College Dublin, Ireland *Bernstein operators based on Chebyshev systems*
11:30–11:55	Maryam Pazouki, Universität Göttingen, Germany *Bases for kernel-based spaces*
11:55–12:20	Stefano De Marchi, University of Padova, Italy *3-dimensional Weakly Admissible Meshes: interpolation and cubature*
12:30	**Lunch**

Afternoon Session:

15:00–15:50	Martin Buhmann, Justus-Liebig-Universität Gießen, Germany *On CS matrix radial basis functions*
16:20–16:45	Franz-Jürgen Delvos, Universität Siegen, Germany *Boolean methods in periodic Hilbert spaces*

Abstracts

Approximation in high dimensions

Peter Binev

There are two prominent approaches to treating high dimensional problems: neural networks and kernel methods, which can be implemented in high dimensions without meshing. In addition to being numerically intensive, both these methods are not multiscale and are hard to adapt locally. Our goal is to develop *alternatives* to neural networks and kernel methods which will mitigate these disadvantages but still be applicable in high space dimensions. To develop a cohesive theory for high dimensional problems requires therefore once again models for data sets which measure information content, now primarily regarding their dimensionality. Adaptive partitioning has been a powerful tool in solving many data processing and computational problems. Any adaptive procedure can be associated with a tree which keeps track of the cells that have been refined at each iteration. However, the standard techniques from adaptive partitioning cannot be implemented directly in high space dimension and thus we introduce the notion of *sparse occupancy trees*. The idea is to relate the adaptive partitions to trees whose nodes correspond to cells which are occupied. A special indexing and ordering of these cells allows storing of all the information about the tree using a number of bits proportional to the space dimension N times the logarithm of the data size. Moreover, by using the tree structure one can efficiently find the bit address of any point $x \in \mathbb{R}^N$. The above structure can be used as a tool to build different algorithms for approximation. In particular, we consider a piecewise linear approximation on simplicial partitions generated over sparse occupancy trees.

One practical realization of this approach is concerned with the problem of *classification*. It can be formulated as finding an unknown function $f : X \to Y$ using (possibly noisy) samples (x_i, y_i) where $y_i \in Y = \{+1, -1\}$ is related to x_i by the law f. Then the problem is to find a set $\Omega \subset X$ that generates the predictor $T_\Omega := 1_\Omega - 1_{\Omega^c}$ such that the risk

$$R(\Omega) := \int_X \mathrm{Prob}(T_\Omega \neq y) d\rho_X$$

is minimized. In case one is allowed to choose any subset Ω of X, the solution is the *Bayes classifier* defined via the set $\Omega^* := \{x : f_\rho(x) > 0\}$ with $f_\rho := \int_Y y \; d\rho(y|x)$ being the regression function and $\rho(y|x)$ is the conditional probability measure on Y with respect to x. However, in practical problems the set Ω should be chosen from a finite family of sets $\mathcal{F}$ using a finite set Z of points. We define the notion of *reliable sets* and use them to identify the building blocks in the adaptive construction of the family $\mathcal{F}$ based on sparse occupancy trees.

Construction of point configurations on hyper spheres

Johann Brauchart

There are a variety of needs for discretization of a (hyper) sphere — interpolation schemes, numerical integration rules, statistical sampling.
Optimization of certain energy functionals yield, for example, uniformly distributed points (E. B. Saff and A. B. J. Kuijlaars [Math. Intelligencer 19:1 (1997)]), quasi-uniformly distributed points with a limit distribution with pre-scribed density (D. P. Hardin, E. B. Saff, J. T. Whitehouse [arXiv:1104.2911v1]), and spherical designs (I. H. Sloan and R. S. Womersley [J. Approx. Theory 159 (2009)]). Explicit constructions utilizing this approach are scarce and one has to rely on numerically obtained low-energy point sets. On the other hand, digital nets and sequences as introduced by H. Niederreiter [Monatsh. Math. 104 (1987)] provide a very efficient method to generate low-discrepancy point sets in the d-dimensional unit cube that are well suited for quasi-Monte Carlo rules. Such and other low-discrepancy configurations can be lifted to the unit sphere by means of an area preserving map.
Finally, we present results for sequences of approximate spherical t-designs that emulate spherical t-designs in the sense that they *approximately* integrate all polynomials of degree less than or equal t and still achieve the asymptotic bounds associated with spherical t-designs.
This talk is based on joint work with Josef Dick, Ian Sloan and Rob Womersley (UNSW) and Ed Saff (Vanderbilt).

On CS matrix radial basis functions

Martin Buhmann

We will discuss radial basis functions with compact support from various classes, both scalar and (in more detail) matrix-valued. Some further examples with global support and parameters will also be mentioned. The main purpose of these new radial functions is in statistical applications where positive definite kernels are usually needed. It is specifically required that these radial functions are not only positive definite in the normal sense but also that their values (matrices) are themselves positive definite.
This is joint work with M. Bevilacqua, D. Daley and E. Porcu.

Kernel based approximation: from data to special functions

Wolfgang zu Castell

Kernel based approximation methods have turned out to provide powerful tools in many fields of application. Although most of the work in the field is dedicated to radial basis functions, many aspects of kernel based methods have found their counterparts in other geometric settings in the meantime.
The central handle from the point of view of applications is the kernel itself. It provides the basic model for similarity used to analyse given data. Therefore, appropriate design of the kernel is an important part of the modeling task to be addressed, making the method work as expected.
In constrast to the ease of motivating the kernel based approach itsef, the aim to derive new kernels often turns out to be surprisingly hard. We will address some of these questions and show how special functions come into play and can lead to solutions.

Restricted spectral radius: non-uniform subdivision schemes

Maria Charina

We use linear programming to construct a C^1 non-uniform butterfly scheme, whose subdivision rules depend on the position of the vertices on a subdivision mesh. The problem of linear programming we solve arises from the restricted spectral radius approach that characterizes the C^k-regularity of stationary subdivision.

Hierarchical Riesz bases for Sobolev spaces on polygonal domains

Oleg Davydov

We present a new construction of Riesz bases for the spaces $H^s(\Omega)$, $1 < s < 5/2$, relying on Lagrange interpolation by C^1 piecewise quadratic polynomials. In contrast to earlier results in [1], an arbitrary triangulation of the polygonal domain Ω can be used, which facilitates the numerical implementation. Similar to [2], mixed Powell-Sabin 6- and 12-splits are employed to generate suitable spline spaces, but the stability range of our wavelets is larger. The results are obtained jointly with Wee Ping Yeo.

[1] O.Davydov and R.Stevenson, Hierarchical Riesz Bases for $H^s(\Omega)$, $1 < s < 5/2$, Constr. Approx. 43 (2005), 365-394.
[2] R.-Q.Jia, S.-T.Liu: C^1 spline wavelets on triangulations. Math. Comput. 77 (2008), 287-312.

Boolean methods in periodic Hilbert spaces

Franz-Juergen Delvos

Babuska introduced the concept of periodic Hilbert spaces in studying optimal approximation of linear functionals. We used these spaces to study the approximation properties of trigonometric interpolation and periodic spline interpolation.
We will investigate approximation by generalized Fourier partial sums constructed by Boolean methods in periodic Hilbert spaces. In particular we will consider bivariate applications.

3-dimensional weakly admissible meshes: interpolation and cubature

Stefano De Marchi

In the paper by Calvi and Levenberg [3], it has been recognized that the so-called *Admissible Meshes (AM)* play a central role in the construction of multivariate polynomial approximation processes on compact sets of $\mathbb{R}^d$. The concept is essentially a matter of polynomial inequalities.
Since for many compacts the computational use of such admissible meshes becomes difficult or even impossible for $d = 2, 3$ already at moderate polynomial degrees, one can consider *Weakly Admissible Meshes (WAM)*, which have lower cardinality than admissible ones.
In the present work, we concentrate on the construction of WAMs for 3-dimensional domains, such as, cones and toroidal sections, with two main applications in mind:

- polynomial interpolation at the so called Approximate Fekete Points (AFP) and/or Discrete Leja Points (DLP) (cf. [1,2]);
- cubature at AFP or DLP.

References:

[1] L. Bos, S. De Marchi, A. Sommariva and M. Vianello, *Computing multivariate Fekete and Leja points by numerical linear algebra*, SIAM J. Num. Anal. 48(5) (2010), 1984–1999.

[2] L. Bos, S. De Marchi, A. Sommariva and M. Vianello, *Weakly Admissible Meshes and Discrete Extremal Sets*, Numer. Math. Theor. Meth. Appl. 4(1) (2011), 1–12

[3] J.-P. Calvi and N. Levenberg, *Uniform approximation by discrete least squares polynomials*, J. Approx. Theory 152 (2008), 82–100.

Optimal fusion frames

Martin Ehler*, Christine Bachoc

Fusion frames enable signal decompositions into weighted linear subspace components and provide efficient reconstruction formulas. We introduce the p-fusion frame potential and verify that its minimizers suppress correlated noise most efficiently. In fact, cubature formulas for Grassmann spaces are minimizers, and, if p goes to infinity, our estimates of the p-fusion frame potential generalize the simplex bound known in coding theory. If the simplex bound is matched, then the subspaces must be equiangular, and we derive an upper bound on the number of equiangular subspaces. The above concepts are also carried over into a probabilistic setting.

Approximation on manifolds

Frank Filbir*, Hrushikesh N. Mhaskar

We will present recent work on approximation of functions defined on (Riemannian) manifolds by so-called networks. A main focus lies on the construction of kernel methods (discrete and continuous) which have an almost optimal degree of approximation. The crucial step in the development of such methods is the construction of well localized kernels. We will demonstrate how to get diffusion polynomial kernels which have a very good localization along the diagonal and relate this results to certain heat kernel estimates.

The presentation is based on joint work with Hrushikesh N. Mhaskar, Department of Mathematics, California State University, Los Angeles, USA.

Image inpainting via analysis-side L_1-minimization

Emily King*, Gitta Kutyniok, Xiaosheng Zhuang

An issue in data analysis is that of incomplete data, for example a photograph with scratches or seismic data collected with fewer than necessary sensors. There exists a unified approach to solving this problem and that of data separation: namely, minimizing the ℓ_1 norm of the analysis (rather than synthesis) coefficients with respect to particular frame(s). There have been a number of successful applications of this method recently. Analyzing this method using the concept of clustered sparsity leads to theoretical bounds and results, which will be presented. Furthermore, necessary conditions for the frames to lead to sufficiently good solutions will be shown.

Sparse FFTs and photoacoustic tomography

Stefan Kunis*, T. Görner, R. Hielscher, I. Melzer

Recently, butterfly approximation schemes have been developed and add a new facet to the field of analysis based fast algorithms. We discuss the idea of these schemes which relies on a sequence of low rank approximations and show in which way these local approximations influence the final accuracy and arithmetic complexity of the method. In case of a fast Fourier transform for sparse data, we give an error estimate and discuss the numerical stability of different variants. Numerical examples and an application to three-dimensional photoacoustic tomography close the talk.

Shearlets and sparse approximation

Gitta Kutyniok

Many important problem classes are governed by anisotropic features such as singularities concentrated on lower dimensional embedded manifolds. While the ability to reliably capture and sparsely represent anisotropic structures is obviously the more important the higher the number of spatial variables is, the principal difficulties arise already in two and three spatial dimensions and even there are yet far from being understood. Five years ago, shearlets were introduced as a means to sparsely encode anisotropic singularities of 2D data in an optimal way, while – in contrast to previously introduced directional representation systems – providing a unified treatment of the continuous and digital world.
In this talk, we will first give a general introduction to the theory of shearlets. Then some very recent results on the construction of compactly supported shearlet in 2D and 3D will be highlighted, in particular, showing that these shearlet frames provide optimally sparse approximations of anisotropic features. Finally, we will discuss applications of shearlet decompositions such as denoising and data separation.

Locally refinable tensor product splines

Tom Lyche

The central idea of isogeometric analysis is to replace traditional Finite Element spaces by NonUniform Rational B-Splines (NURBS) to provide accurate shape description and elements with higher degrees and smoothness. Since the introduction of the idea in 2005 by Tom Hughes and co-workers, promising results have been obtained documenting its potential. However, it has also been demonstrated that NURBS do not support the local refinement needed in efficient finite element analysis. To overcome this deficiency the use of T-splines as introduced by Sederberg et al is a promising alternative. T-splines are tensor product B-splines on a quadrilateral mesh with T-joins, called a T-mesh. A special case is quadrilateral hierarchical meshes. There are some unresolved problems with T-splines. For example, it has been observed that refinement along a diagonal in a T-mesh can lead to non-local refinement and rational basis functions can occur. To overcome some of these problems an alternative called LR-splines will be considered. Here local refinement of tensor product B-splines is specified as a sequence of inserted line segments parallel to the knot lines. Affected B-splines are split into two new B-splines using univariate knot insertion. We obtain a special case of a T-mesh, here named an LR-mesh. On the LR-mesh we obtain a collection of tensor product B-splines which form a partition of unity. The approach applies equally well in dimensions higher than two. In the bivariate case this collection of B-splines spans the full piecewise polynomial space on the LR-mesh and is independent of the order in which the segments are inserted.

Non standard discretization techniques in isogeometric analysis

Carla Manni

Isogeometric Analysis is an emerging framework for analysis of problems governed by partial differential equations. The term *isogeometric* is due to the fact that the solution space for dependent variables is represented in terms of the same functions which represent the geometry.
Isogeometric Analysis developed so far is mainly based on NURBS, and has revealed to be an effective tool as in this way it is possible to fit exact geometries at the coarsest level of discretization and eliminate geometry errors from the very beginning. Moreover, the approach can also manage approximating functions with high order smoothness. This is also regarded as a benefit.
Nevertheless rational representations present several drawbacks. This motivated the introduction in the context of CAGD of some possible alternatives to the rational model.
In this talk we review the use of some possible alternatives to NURBS in Isogeometric Analysis, with special focus on the problem of local refinements.

A first constructive characterisation of all piecewise Chebyshevian splines suitable for approximation or design

Marie-Laurence Mazure

By *piecewise Chebyshevian splines* we mean splines with sections in different Extended Chebyshev spaces and with possibly connection matrices at the knots. In this general difficult framework (first considered by P.J. Barry in 96), existence of blossoms is known as a beautiful and powerful characterisation for a spline space to be suitable for Design (or Approximation as well). Nevertheless, beyond the analogue of the cubic spline case, up to now it remained a theoretical characterisation, not a really constructive one.
In the present work, we give a surprisingly simple description of ALL piecewise Chebyshevian splines which do possess blossoms. It is a contructive characterisation in two ways:

1- at last we are now able to build in a simple way exactly all suitable piecewise Chebyshevian spline spaces, in any dimension and with any multiplicities;

2- a space of piecewise Chebyshevian splines being given, we are now able to answer the question "is it or not suitable for design?"

Let us mention that the answer to the latter question is made possible by a recently published work in which we gave a constructive way to build all systems of weight functions which can be associated with a given Extended Chebyshev space on a closed bounded interval.

The multivariate moment problem

H. M. Möller

Let $\mathcal{P} := \mathcal{R}[x_1, \ldots, x_n]$ and $\mathcal{P}^*$ its algebraic dual. The multivariate moment problem concerns the characterisation of all linear functionals $L \in \mathcal{P}^*$ having a representation

$$L(x^i) = \int x^i \; d\mu(x) \text{ for all } i \in \mathcal{N}_0^n \ .$$

Here μ denotes a positive Borel measure on $\mathcal{R}^n$. Haviland (1935) showed that the set of these functionals is

$$K_+^* = \{L \in \mathcal{P}^* \mid \forall f \in \mathcal{P} : (\forall \xi \in \mathcal{R}^n : f(\xi) \geq 0) \;\; \Rightarrow \;\; L(f) \geq 0\} \ .$$

The question remained open, how to describe the set

$$\Sigma^* = \{L \in \mathcal{P}^* \mid \forall \{f_1, \ldots, f_s\} \subset \mathcal{P} : L(\sum_{i=1}^{s} f_i^2) \geq 0\} \ .$$

We show $K_+^* = \Sigma^*$. This allows for instance a characterisation of real ideals by integrals and to solve the inverse problem of numerical integration.

Divided differences of multivariate implicit functions

Georg Muntingh

Under general conditions, the equation $g(x_1, \ldots, x_q, y) = 0$ implicitly defines y locally as a function of $x_1, \ldots, x_q$. In this talk we present a formula that expresses divided differences of y in terms of divided differences of g, generalizing a recent formula for the case where y is univariate. The formula involves a sum over a combinatorial structure whose elements can be viewed either as polygonal partitions or as planar trees. Through this connection we indicate how to arrive at a simpler formula for derivatives.

On error estimates of cubature formulas exact for Haar polynomials

Mikhail Noskov

The talk contains the results of the research of error functionals norms of cubature formulas exact for Haar polynomials in Sobol' spaces. The upper and lower estimates for the norms in the spaces H_α and S_p, including the estimates for weight cubature formulas, are obtained.

Ortho-projectors onto linear spline spaces over arbitrary triangulations: L_∞-bounds

Peter Oswald

We will discuss the L_∞ boundedness of the L_2-orthoprojection onto spaces of linear splines over arbitrary triangulations in two dimensions. Emphasis is on criteria guaranteeing uniform boundedness of the L_∞ bounds, and (motivated by needs in the analysis of the finite element method) the case the triangulation is obtained from a rectangular tensor-product partition by further subdivision.

Bases for kernel-based spaces

Maryam Pazouki*, Robert Schaback

Practical experience shows that the coefficients of Kernel-based interpolants in terms of the basis of translates of the kernel are unstable due to the bad condition of the kernel matrix. However, due to results of S. De Marchi and R. Schaback, the resulting interpolant is stable in function space. This implies that there must be better bases than the standard one. This work provides a variety of different bases based on factorizations of the kernel matrix. These bases differ in their stability, orthogonality, adaptivity, duality, and computational efficiency properties. Special emphasis is given to the "Newton" basis arising from a pivoted Cholesky factorization. It turns out to be stable and computationally cheap while being orthonormal in the "native" Hilbert space of the kernel. There are efficient adaptive algorithms for calculating the Newton basis along the lines of Orthogonal Matching Pursuit.

Local refinements in isogeometric analysis with generalized B-splines

Francesca Pelosi*, Carla Manni, Maria Lucia Sampoli

Local refinement is one of the key issues in Igogeometric Analysis, the novel approach for solving PDE's problems. In order to properly address such problem, several alternative representations to classical tensor-product B-splines, have been considered. One promising aones is given by Hierachical B-splines recently discussed in Jüttler et al. (2011), where B-spline (or NURBS) basis functions have been considered as the main "building block". In this talk we shall present a first study on how using Generalized B-splines instead could generate a new space of Hierachical splines with the same nice properties but with a more flexibility.

Compactly supported correlations: a statistical view

Emilio Porcu

Estimation of covariance parameters requires a considerable cost from the computational viewpoint, in particular when dealing with massive datates. Such a problem is even bigger in the case of vector-valued random fields. We propose criteria for the construction of mappings from R^d to $p \times p$ matrices with the requirement of being positive definite. We propose some correlation matrix-valued functions whose members have different compact support and different level of smoothness. We illustrate the features of such models through simulation as well as data analysis

Methods of the exact joint spectral radius computation

Vladimir Protasov

This is well known that the problem of computing or of estimating the joint spectral radius of matrices plays a crucial role in the study of wavelets, refinable functions, and subdivision algorithms in approximation theory. We discuss possible approaches to this problem and present a method of exact computation of the joint spectral radius, which is applicable for general matrices in relatively high dimensions. We discuss some applications of this method to refinable functions and subdivision algorithms.

Compressive sensing, structured random matrices and recovery of functions in high dimensions

Holger Rauhut

Compressive sensing is a recent paradigm in signal processing and sampling theory that predicts that sparse signals can be recovered from a small number of linear and non-adaptive measurements using convex optimization or greedy algorithms. Quite remarkably, all good constructions of the so called measurement matrix known so far are based on randomness. While Gaussian random matrices provide optimal recovery guarantees, such unstructured matrices are of limited use in applications. Indeed, structure often allows to have fast matrix vector multiplies. This is crucial in order to speed up recovery algorithms and to deal with large scale problems. The talk discusses models of structured random matrices that are useful in certain applications, and presents corresponding recovery guarantees. An important type of structured random matrix arises in connection with sampling sparse expansions in terms of bounded orthogonal systems (such as the Fourier system). Applications to function recovery in high dimensions will be given. The second type of structured random matrices to be discussed are partial random circulant matrices. These model convolution with a random sequence followed by subsampling. A third type of structure arises from time-frequency analysis. We present in particular recent results on the restricted isometry constants of related random matrices.

Bernstein operators based on Chebyshev systems

Hermann Render

In the last decade generalized Bernstein bases have been extensively discussed in the framework of extended Chebyshev spaces. In this talk, based on joint work with J. Aldaz (Universidad Autonoma de Madrid) and O. Kounchev (Bulgarian Academy of Sciences), we consider generalized Bernstein operators B_n fixing a strictly positive function f_0, and a second function f_1 such that f_1/f_0 is strictly increasing. We give an inductive criterion for existence: suppose that $U_n \subset U_{n+1}$ are extended Chebyshev spaces over $[a, b]$ and suppose that there exists a Bernstein operator $B_n : C[a, b] \to U_n$ with strictly increasing nodes, fixing $f_0, f_1 \in U_n$. Then there exists a Bernstein operator $B_{n+1} : C[a, b] \to U_{n+1}$ with strictly increasing nodes, fixing f_0 and f_1. In particular, if $f_0, f_1, ..., f_n$ is a basis of U_n such that the linear span of $f_0, .., f_k$ is an extended Chebyshev space over $[a, b]$ for each $k = 0, ..., n$, then there exists a Bernstein operator B_n with increasing nodes fixing f_0 and f_1. Moreover, we present shape preserving properties of the generalized Bernstein operator and we give a sufficient criterion for a function f such that the inequality $B_n f \geq B_{n+1} f \geq f$ holds.

In the second part of the talk we shall discuss multivariate Bernstein-type operators for functions defined on the unit ball in euclidean space with some specific properties. The construction is based on the Fourier-Laplace series of the function and univariate Bernstein operators acting on the Fourier-Laplace coefficients. This leads to the construction of Bernstein-type operators fixing all harmonic polynomials on the unit ball.

Divided differences by interpolation

Tomas Sauer

There are many generalizations of divided differences to the multivariate case, depending on which of the many properties of the univariate differences one wishes to extend. One possibility, pursued by de Boor, is to extend simplex spline integrals, another one is to define divided differences as the leading coefficients or leading forms of an interpolation polynomial. This latter approach, however, requires to clarify which concept of multivariate polynomial interpolation one wishes to pursue. The talk considers divided differences based on monomial minimal degree interpolation spaces which indeed generalize a lot of properties of the univariate divided difference. A recurrence relation, connections to simplex splines and even a product formula will be given.

Partially, this is joint work with Jesús Carnicer from Zaragoza.

Splines on meshes with hanging vertices

Larry Schumaker*, Lujun Wang

A constructive theory is developed for polynomial spline spaces defined on mixed meshes consisting of triangles and rectangles. These meshes include triangulations with hanging vertices as well as T-meshes. Such meshes are useful in both CAGD and the FEM method. In addition to dimension formulae, explicit basis functions are constructed, and their support and stability properties are discussed. The approximation power of the spaces is also treated.

Orthogonal spline-projector and related questions

Alexei Shadrin

We will make an overview of some problems related to de Boor's 1972 conjecture (and its subsequent proof in 2001) on the max-norm boundedness of the orthogonal spline-projector of degree k. We start with the problems which were initial points of interest, namely construction of orthogonal bases in the spaces of continuous and r times differentiable functions (Ciesielski, Domsta), error of spline interpolation (de Boor, Subbotin), Galerkin approximation to solutions of boundary value problems (Douglas, Dupont, Wahlbin). We then describe several alternative approaches that were used in various particular cases (e.g., quadratic and cubic splines), as well as the key ideas of the general proof. Finally, we discuss developments that happened after the proof. They include: boundedness of the spline interpolation operator in C^{k-1} (Volkov, note that de Boor's conjecture is equivalent to C^k-boundedness), $\sqrt{k}$-bound for the actual value of the max-norm for splines with low smoothness (Foucart, Kayumov), rigorous extension to splines on infinite and periodic knot-sequences (de Boor), a counterexample for the linear splines on triangulations (Oswald), and some other.

Normalized hierarchical B-splines

Hendrik Speleers*, Carlotta Giannelli, Bert Jüttler

The key requirement of an effective mesh refinement algorithm is to provide a local and adaptive procedure which enables to refine the underlying geometry so that only specific regions of the parameter domain are influenced. This kind of refinement is naturally supported by the hierarchical spline model, where different levels of details are identified through a hierarchy of B-splines. The hierarchical model controls the locality of the refinement through an adaptive procedure which selects a suitable set of basis functions from different hierarchical levels – the hierarchical B-spline basis. The construction of a hierarchical B-spline basis can be suitably modified in order to define locally supported basis functions that form a partition of unity. We will show that this property can be obtained by reducing the support of basis functions defined on coarse grids, according to finer levels in the hierarchy. This truncation process not only decreases the overlapping of basis supports, but it also improves the stability properties of the corresponding hierarchical basis.

Penalized frame potential for clustering short time series

Tobias Springer*, Katja Ickstadt, Joachim Stöckler

The clustering of short time series is a statistical problem occuring in the analysis of microarray data. In this talk, we give a short geometric interpretation of the commonly used dissimilarity measure

$$\mathrm{dis}(p,q) = 1 - \rho(p,q)$$

with Bravais-Pearson correlation coefficient ρ between the time series $p, q \in \mathbb{R}^{d+1}$. Based on an approach by Benedetto, Czaja and Ehler ([1]) for sparse representations of retinal data, we propose to minimize the Penalized Frame Potential

$$\min_{\Theta \subset \mathcal{S}^{d-1}} \mathrm{PFP}_\lambda(\Theta, Y) = \min_{\Theta \subset \mathcal{S}^{d-1}} \frac{\lambda\, d}{m^2}\,\mathrm{TFP}(\Theta) \,+\, (1-\lambda) m\, \mathrm{P}(\Theta, Y), \quad \lambda \in [0,1],$$

including the (Total) Frame Potential TFP, a data-dependent penalty term P and given time series data Y. Different penalty terms P will be presented with results of the application to simulated and real data with comparison to the STEM algorithm in [2].

References:

[1] J.J. Benedetto, W. Czaja, M. Ehler: Frame potential classification algorithm for retinal data, Springer Proc. Series: IFMBE, 26$^{\text{th}}$ Southern Biological Engineering Conference (2010).

[2] J. Ernst, G.J. Nau, Z. Bar-Joseph: Clustering short time series gene expression data, Bioinformatics, Vol. 21, 2005.

Quadrature rules, discrepancies and their relations to halftoning on the torus and the sphere

Gabriele Steidl*, Manuel Gräf, Daniel Potts

The stippling technique places black dots such that their density gives the impression of tone. This is the first paper that relates the distribution of stippling dots to the classical mathematical question of finding 'optimal' nodes for quadrature rules. More precisely, we consider quadrature error functionals on reproducing kernel Hilbert spaces (RKHSs) with respect to the quadrature nodes and suggest to use optimal distributions of these nodes as stippling dot positions. Interestingly, in special cases, our quadrature errors coincide with discrepancy functionals and with recently proposed attraction-repulsion functionals. Our framework enables us to consider point distributions not only in R^2 but also on the torus T^2 and the sphere S^2. For a large number of dots the computation of their distribution is a serious challenge and requires fast algorithms. To this end, we work in RKHSs of bandlimited functions, where the quadrature error can be replaced by a least squares functional. We apply a nonlinear conjugate gradient (CG) method on manifolds to compute a minimizer of this functional and show that each step can be efficiently realized by nonequispaced fast Fourier transforms. We present numerical stippling results on S^2.

On the dimension of triangular spline spaces

Nelly Villamizar*, Bernard Mourrain

The space of spline functions attached to a subdivided planar domain plays an important role in CAGD and has been considered recently for isogeometric analysis applications. Using homological techniques, we found a lower and an upper bound to the dimension of this spline space, which are more general and give better approximations to the exact value of the dimension than the already existing ones. The formulas allow us to recover the known cases where the lower and upper bounds coincide and also give us some insight on the type of hierarchical subdivision strategy to employ in order to keep this property. The results can be extended to spline spaces on 3-dimensional complexes and applied to any rectilinear subdivision of a polygonal domain, and to mixed splines, which are splines where the order of smoothness may differ on the various edges.

Scalar multivariate subdivision schemes with scaling factor > 2 and box splines

Georg Zimmermann

It is well known that in univariate scalar subdivision with scaling factor $m = 2$, the order of polynomial reproduction equals the multiplicity of -1 as a zero of the mask symbol. This means that in the ring of Laurent polynomials, its mask symbol is contained in the ideal

$$\mathcal{I}^k = \left\langle (1+z)^k \right\rangle ,$$

i. e., the ideal generated by the mask symbol of the cardinal B-spline of order k. This is still true for integer scaling factors $m > 2$. The cardinal B-splines are refinable for any such m, their mask symbols are given by

$$(1 + z + z^2 + \cdots + z^{m-1})^k / m^{k-1} = m \left(\frac{1 - z^m}{m\,(1-z)} \right)^k ,$$

and a univariate scalar subdivision scheme with scaling factor m has order of polynomial reproduction at least k if and only if its mask symbol is contained in the ideal

$$(\mathcal{I}^{[m]})^k = \left\langle (1 + z + \cdots + z^{m-1})^k \right\rangle .$$

Similar results have been shown in [Charina, Conti, Jetter, Zimmermann 2011] for the case of bivariate scalar subdivision with scaling matrix $2\,I$: the corresponding ideals are generated by the mask symbols of a certain family of box splines of the appropriate order.
We extend these results to subdivision schemes with scaling matrix $3\,I$.